STATISTICAL MECHANICS AND STRANGE TRANSPORT

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In recent years Continuous Time Random Walks (CTRW) and Fractional Differential Equations (FDE) have proved to be extremely successful modelling techniques for describing a wide range of applications for which standard diffusive transport is found experimentally to be inadequate. Yet there does not exist a complete justification of these concepts based on first principles of mechanics. A tractable starting point is provided by a "semi-dynamical"approach, based on a V-Langevin equation: an equation of motion of Newtonian (or Hamiltonian) type for a tracer particle moving in presence of a random potential.Associated with it there is a "hybrid kinetic equation" (HKE) for the (stochastic) distribution function $f(\mathbf{x},t)$ of the positions. By standard methods of statistical mechanics an equation of evolution of the ensemble-average of this function, called the "density profile" $n(\mathbf{x},t) = \langle f(\mathbf{x},t)\rangle$ is derived. The latter is, however, not closed because of its non-local character: in its right hand side appears, under an ensemble-average, the density profile evaluated at the fluctuating position of the particle, together with other fluctuating quantities. The usual "local approximation" provides a good description of normal diffusive processes, but is inadequate for strange transport (sub- or supra-diffusive).

Recently, Sanchez et al. (Phys. Rev. Lett., to be published) used an elegant method for overcoming the non-locality difficulty, based on functional integration techniques applied to the fluctuating particle trajectories (supposed to be self-similar) in order to derive a closed non-local equation for the density profile. Under special assumptions this equation can be reduced to a FDE.

In the present work we use a quite different approach, based on an analysis of the various types of propagators appearing in the treatment of the HKE. We introduce a non-local extension of an approximation similar to the Corrsin factorization assumption of turbulence theory. The result is a *non-Markovian and non-local*, formally linear equation, in which the rate of change of the density profile $n(\mathbf{x},t)$ at point $\mathbf{x}$ and time t is related to the values of this function at neighbouring points $\mathbf{x} + \mathbf{r}$ and at past times $t - \tau$. Its structure is similar, but not identical (because of different approximations) with the equation of Sanchez et al. On the other hand, it can be shown that there exists an "equivalent" CTRW. The transition probability in the Montroll-Shlesinger equation describing the latter is related to the Eulerian velocity autocorrelation and to the ensemble-averaged propagator of the HKE $\langle G(\mathbf{x},t|\mathbf{x}',t')\rangle$. Under certain special assumptions on the form of the latter two quantities (such as self-similar power-law forms), the equation for the density profile can be reduced to a fractional differential equation.

When viewed from a more general point of view, the equation of evolution for $n(\mathbf{x},t)$ is readily transformed into an equation for the average propagator $\langle G(\mathbf{x},t|\mathbf{x}',t')\rangle$. The latter provides, as usual, the solution of the Dirichlet problem of the former equation for arbitrary initial condition $n(\mathbf{x},0)$. Besides its non-Markovian and non-local character, this equation appears to be explicitly *nonlinear*. Thus, - not surprisingly - even in this simplest "non-local Corrsin-like" approach, one is faced with the complexity of a nonlinear process. A self-consistent theory of strange transport should therefore involve adequate approximation methods (such as renormalization techniques) for treating this equation.

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