Subdiffusion and chemical reactions in a porous medium

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We study a system with subdiffusion of particles of spices A and B which chemically react to produce an inert product. The system is described by the non-linear equations with the Riemann-Liouville fractional time derivative

$$\frac{\partial^\alpha}{\partial t^\alpha} C_i(x,t) = D_{\alpha i} \frac{\partial^2}{\partial x^2} C_i(x,t) - k C_A(x,t) C_B(x,t), \tag{1}$$

where $i = A, B$, C_i denotes concentration of the diffusing particles of spice i, $D_{\alpha i}$ - its subdiffusion coefficient. Using the quasistatic approximation [1] we find the time evolution of some system's characteristics such as the position of the reaction front, the width of the reaction zone and the depletion one etc.

In particular, we consider the system with one static reactant B ($D_B = 0$) [2]. The progress of carious lesion in the tooth enamel is considered as an example of such a situation. The caries is caused by some organic acid molecules which react with the static hydroxyapatite (which is a main component of the enamel). Since the enamel is a porous medium, there is the subdiffusion of large acid molecules inside it. Comparing the theoretical results with the experimental ones we find the values of the subdiffusion parameter α for the substances under considerations.

References

[1] T. Kosztołowicz and K. D. Lewandowska *Time evolution of the reaction front in a subdiffusive system*, cond-mat/0603139 (2006).

[2] T. Kosztołowicz and K. D. Lewandowska *Time evolution of the reaction front in the system with one static and one subdiffusive reactant*, Acta Phys. Pol. B (in press).