Fractional sub-diffusion equation - a mathematical approach for quantifying nematode dispersal in homogeneous environments

Simona Hapca, John Crawford and Iain Young

Simbios Centre, University of Abertay, Dundee, UK

Abstract

We consider correlated random walk model in two dimensions for simulating the movement of the slug parasitic nematode *P.hermaphrodita*. The model incorporates turning angle distribution and speed distribution of the movement that are obtained from direct observations of individual's behaviour. Unlike a simple random walk, in which the successive steps are independent, in our model we identify and express strong correlations between the turning angles and between the steps length. Finally we find that these correlated random walks can be described in terms of a fractional sub-diffusion equation as follows:

$$\partial_t p_t(x) = {}_0D_t^{1-\alpha} K_\alpha \Delta_x p_t(x) \quad \text{where} \quad {}_0D_t^{1-\alpha} f(t,x) = \frac{1}{\Gamma(\alpha)} \frac{\partial}{\partial t} \int_0^t \frac{f(s,x)}{(t-s)^{1-\alpha}} ds$$

is the Riemann-Liouville operator defined for any $0 < \alpha < 1$ and p is the probability density function quantifying the nematode dispersal.

Strong memory effects on the level of the probability distribution function characterize the associated stochastic process i.e., unlike a Markov process, the now-state of the system depends on the entire history.

Key words: correlated random walk, conditional probabilities, fractional sub-diffusion equation, stochastic processes, long time memory, mean square displacement, turning angle distribution, speed distribution.