Fractional diffusion models of anomalous transport: theory and applications

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The importance of anomalous diffusion is well recognized in many areas including physics, chemistry, engineering, and biology. Despite this, anomalous transport is not well understood, and there is the need to develop mathematical models with predictive capabilities. Relatively recently, tools and ideas from fractional calculus have been proposed to construct such models. Fractional derivatives are integro-differential operators that provide a unifying framework to model key aspects of anomalous transport including: non-locality, non-Markovian (memory) effects, non-Gaussian (Levy) processes, and non-diffusive scaling. In this talk we present fractional diffusion models of anomalous transport in unbounded and finite-size domains, and discuss applications to plasma physics and non-linear reaction-diffusion systems.

The plasma physics applications are centered on the study of anomalous transport in high temperature plasmas. This problem is of critical importance for the success of the nuclear fusion program. Nuclear fusion is the process that stars use to produce energy, and the goal of the fusion program is to exploit the same mechanism to produce controlled thermonuclear energy. Transport in fusion plasmas is in general anomalous and standard diffusive models have limited applicability. We show that fractional diffusion models provide a viable alternative to describe transport in this system. In particular, we propose and test fractional diffusion models describing transport of tracers in plasma turbulence in the presence of coherent structures. Also, we propose a finite-size domain fractional transport model to describe anomalous heat transport. It is shown that the proposed models reproduce the basic anomalous transport phenomenology including non-local effects, fast propagation phenomena, non-Fickian transport, and non-Gaussian scaling

In the second part of this presentation we discuss the application of fractional diffusion to reaction-diffusion systems in the presence of anomalous transport. In particular, we present an analytical and numerical study of the fractional Fisher-Kolmogorov equation describing front propagation in systems with anomalous diffusion caused by Levy flights. We show that fractional diffusion leads to exponential fast propagation of fronts and to the development of algebraic tails.