Escape of a Lévy Particle from a Potential Well

Alexei Yu. Slyusarenko
Karazin Kharkov National University, 61108, Kurchatova Av., 31, Kharkov, Ukraine

We consider the motion of a Lévy particle (that is the particle being under the influence of an α-stable external random force) in a potential well. Our two main purposes are the evaluation of (i) the mean escape time from the well (the generalized Kramers' problem), and (ii) the escape time probability density as a function of time. It is well-known that the classical problem assumes the external force to have a Gaussian statistics [1]. However, after revelation of Lévy processes in nature [2] the Kramers' problem is worth investigating for the broader class of α-stable Lévy distributions with the Lévy index α, $0<\alpha<2$. The reason for this lies in different applications, e.g. in stochastic climate dynamics [3], single-molecule physics [4], engineering [5] etc.

We use the method of numerical integration of an overdamped Langevin equation,

$$\frac{dx(t)}{dt} = -\frac{1}{m\gamma}\frac{dU(x)}{dx} + D^{1/\alpha}\xi_\alpha(t)\ ,$$

where $x(t)$ is particle's coordinate; m is its mass; γ is viscosity constant; $U(x)$ is the external potential; α is the Lévy index; $\xi_\alpha(t)$ is α-stable random force; D is its intensity. The calculations show an extraordinary behavior of the mean escape time $T_c(\alpha, D)$ as a function of noise intensity: in contrast to the well-known exponential law for the Gaussian random force it obeys power law for the Lévy random force at small noise intensities, that is, small D's: $T(\alpha, D) = C(\alpha)/D^{\mu(\alpha)}$.

The preliminary results were given in [6] for α ranging between 1 and 2, however, in the present work simulations are performed with better precision and for two potential profiles in the whole domain of the Lévy indices.

The second issue of the work is the numerical simulation of the probability of leaving the potential well as a function of time. For all Lévy indices α it decays exponentially.

Besides these simulation results, the different analytical approaches to the problem are discussed.

[1] M.I. Freidlin, A.D. Wentzell, *Random perturbations of dynamical systems,*
Grundlehren der Mathematischen Wissenschaften (New York: Springer), **260** (1998).
[2] M.F. Shlesinger, G.M. Zaslavsky, U. Frisch (Eds.) *Lévy flights and related topics in physics,* Lecture notes in physics, **450**, 51 (1995).
[3] P.D. Ditlevsen, Phys. Rev. E, **60**, 172 (1999).

[4] C. Bustamante, J.C. Macosko, G.J.L. Wuite, Nature Reviews (Mol. Cell Bio.), **1**, 130 (2000).
[5] C.L. Nikias, M. Shao, *Signal Processing with Alpha- Stable Distributions and Applications* New York: John Wiley & Sons, 158 (1999).
[6] A.V. Chechkin, V.Yu. Gonchar, J. Klafter, R. Metzler, Europhysics Letters, **72**, 348 (2005).